

解答例は次のとおりです（ C は積分定数）．自分で何度も計算を真似て確かめることが大切です．また、この最終結果を鵜呑みにするのもよくありません．これを見てじっくりと考えてください．また結果の中にある $\tan^{-1} x$ は $\tan x$ の逆関数です．

$$\begin{aligned}
 (1) \int \frac{x+2}{x^4-1} dx &= \int \frac{x}{x^4-1} dx + 2 \int \frac{dx}{x^4-1} \\
 &= \frac{1}{2} \int \frac{(x^2)'}{(x^2)^2-1} dx + \int \frac{dx}{x^2-1} - \int \frac{dx}{x^2+1} \\
 &= \frac{1}{2} \int \frac{dt}{t^2-1} + \frac{1}{2} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1} - \int \frac{dx}{x^2+1} \\
 &= \frac{1}{4} \int \frac{dt}{t-1} - \frac{1}{4} \int \frac{dt}{t+1} + \frac{1}{2} \int \frac{dx}{x-1} - \frac{1}{2} \int \frac{dx}{x+1} - \int \frac{dx}{x^2+1} \\
 &= \frac{1}{4} \log|t-1| - \frac{1}{4} \log|t+1| + \frac{1}{2} \log|x-1| - \frac{1}{2} \log|x+1| - \tan^{-1} x + C \\
 &= \frac{1}{4} \log \left| \frac{t-1}{t+1} \right| + \frac{1}{2} \log \left| \frac{x-1}{x+1} \right| - \tan^{-1} x + C \\
 &= \frac{1}{4} \log \left| \frac{x^2-1}{x^2+1} \right| + \frac{1}{4} \log \frac{(x-1)^2}{(x+1)^2} - \tan^{-1} x + C \\
 &= \frac{1}{4} \log \left| \frac{(x-1)(x+1)(x-1)^2}{(x^2+1)(x+1)^2} \right| - \tan^{-1} x + C \\
 &= \frac{1}{4} \log \left| \frac{(x-1)^3}{(x^2+1)(x+1)} \right| - \tan^{-1} x + C \\
 &= \frac{1}{4} \log \frac{(x-1)^4}{|x^4-1|} - \tan^{-1} x + C
 \end{aligned}$$

$$\begin{aligned}
(2) \int \frac{x^3 - 3x^2 - 4x - 20}{x^4 - 16} dx &= \int \frac{x^3}{x^4 - 16} dx - 3 \int \frac{x^2}{x^4 - 16} dx - 4 \int \frac{x}{x^4 - 16} dx - 20 \int \frac{dx}{x^4 - 16} \\
&= \frac{1}{4} \int \frac{(x^4 - 16)'}{x^4 - 16} dx - \frac{3}{2} \int \frac{dx}{x^2 + 4} - \frac{3}{2} \int \frac{dx}{x^2 - 4} - 2 \int \frac{(x^2)'}{(x^2)^2 - 16} dx - \frac{5}{2} \int \frac{dx}{x^2 - 4} + \frac{5}{2} \int \frac{dx}{x^2 + 4} \\
&= \frac{1}{4} \log|x^4 - 16| + \int \frac{dx}{x^2 + 4} - 4 \int \frac{dx}{x^2 - 4} - 2 \int \frac{dt}{t^2 - 16} \\
&= \frac{1}{4} \log|x^4 - 16| + \frac{1}{2} \int \frac{\left(\frac{x}{2}\right)'}{\left(\frac{x}{2}\right)^2 + 1} dx - \int \frac{dx}{x-2} + \int \frac{dx}{x+2} - \frac{1}{4} \int \frac{dt}{t-4} + \frac{1}{4} \int \frac{dt}{t+4} \\
&= \frac{1}{4} \log|x^4 - 16| + \frac{1}{2} \tan^{-1} \frac{x}{2} - \log \left| \frac{x-2}{x+2} \right| - \frac{1}{4} \log \left| \frac{t-4}{t+4} \right| + C \\
&= \frac{1}{4} \log|x^4 - 16| + \frac{1}{2} \tan^{-1} \frac{x}{2} - \frac{1}{4} \log \left| \frac{(x-2)^5}{(x+2)^3(x^2+4)} \right| + C \\
&= \frac{1}{4} \log \frac{(x+2)^4(x^2+4)^2}{(x-2)^4} + \frac{1}{2} \tan^{-1} \frac{x}{2} + C \\
&= \frac{1}{2} \log \frac{(x+2)^2(x^2+4)}{(x-2)^2} + \frac{1}{2} \tan^{-1} \frac{x}{2} + C
\end{aligned}$$

$$\begin{aligned}
(3) \int x \cos x \sin x dx &= \frac{1}{2} \int x \sin 2x dx \\
&= -\frac{1}{4} x \cos 2x + \frac{1}{4} \int \cos 2x dx \\
&= -\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C
\end{aligned}$$