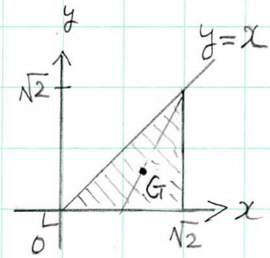
17. 面積1の領域と、その重心の座標

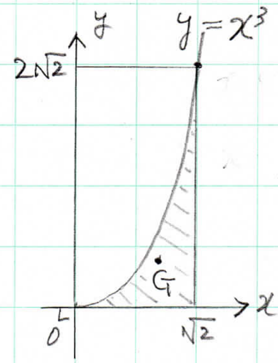
(1) $S=\frac{1}{2}(\sqrt{2})^2=1$



$G(g_1, g_2)$ とすると

$$g_1=\frac{2\sqrt{2}}{3}\fallingdotseq 0.94 \quad , g_2=\frac{\sqrt{2}}{3}\fallingdotseq 0.47$$

(2) $S=\int_0^{\sqrt{2}}x^3\,dx=\left[\frac{1}{4}x^4\right]_0^{\sqrt{2}}=1$



$$V_x=\pi\int_0^{\sqrt{2}}y^2\,dx=\pi\int_0^{\sqrt{2}}x^6\,dx=\pi\left[\frac{1}{7}x^7\right]_0^{\sqrt{2}}$$

$$=\frac{\sqrt{2}^7}{7}\pi=\frac{8\sqrt{2}}{7}\pi$$

$$\therefore g_2=\frac{V_x}{2\pi}=\frac{4\sqrt{2}}{7}\fallingdotseq 0.81$$

$G(g_1, g_2)$

$\frac{dy}{dx}=3x^2$ より

$dy=3x^2\cdot dx$

x	$0 \to \sqrt{2}$
y	$0 \to 2\sqrt{2}$

$$V_y=\pi\sqrt{2}^2\cdot 2\sqrt{2}-\pi\int_0^{2\sqrt{2}}x^2\,dy$$

$$=4\sqrt{2}\pi-\pi\int_0^{\sqrt{2}}x^2\cdot 3x^2\,dx$$

$$=4\sqrt{2}\pi-\pi\cdot\left[\frac{3}{5}x^5\right]_0^{\sqrt{2}}=\frac{8\sqrt{2}}{5}\pi$$

$$\therefore g_1=\frac{V_y}{2\pi}=\frac{8\sqrt{2}}{5}\pi\times\frac{1}{2\pi}=\frac{4\sqrt{2}}{5}\fallingdotseq 1.13$$

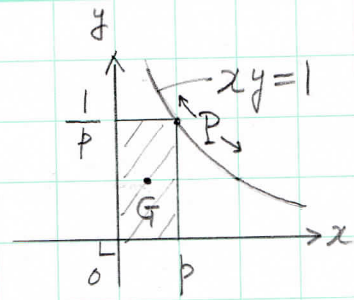
※ パップス、ギュルダンの定理を用いる場合、

V_x：x軸回転体積、その時の重心移動距離 $2\pi g_2$

V_y：y 〃 、 〃 $2\pi g_1$

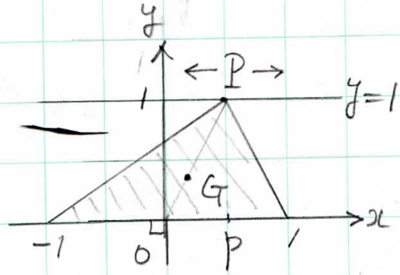
従って $g_1=\frac{V_y}{2\pi}$, $g_2=\frac{V_x}{2\pi}$ となっている。

(3) $S = p \times \frac{1}{p} = 1$



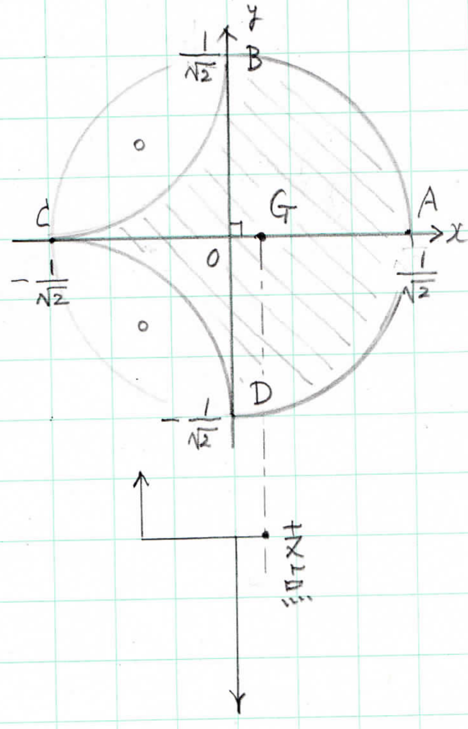
$g_1 = \frac{p}{2}$, $g_2 = \frac{1}{2p}$

(4) $S = \frac{1}{2} \times 2 \times 1 = 1$



$g_1 = \frac{p}{3}$, $g_2 = \frac{1}{3}$

(5) S = □ABCDと同じ $= 1$



Gを支点として。

$$\frac{\pi}{2} g_1 = \left(\frac{\pi}{2} - 1\right) \cdot \left(g_1 + \frac{1}{2\sqrt{2}}\right)$$

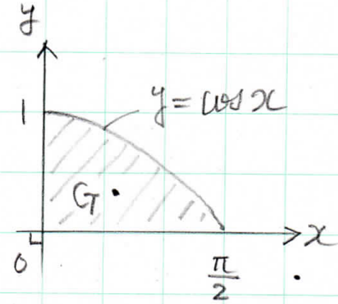
$$g_1 = \frac{\pi - 2}{2} \times \frac{1}{2\sqrt{2}} = \frac{\pi - 2}{4\sqrt{2}}$$

$$= \frac{\sqrt{2}(\pi - 2)}{8} \fallingdotseq 0.20$$

$$g_2 = 0$$

(6)

$$S = \int_0^{\frac{\pi}{2}} \cos x \, dx = \left[\sin x\right]_0^{\frac{\pi}{2}} = 1$$



$$V_x = \pi \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$= \pi \int_0^{\frac{\pi}{2}} \frac{1}{2}(1 + \cos 2x) \, dx$$

$$= \frac{\pi}{2} \left[x + \frac{1}{2} \cdot \sin 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{\pi}{2}\left(\frac{\pi}{2} + \frac{1}{2} \cdot \sin \pi\right) = \frac{\pi^2}{4}$$

$$\therefore g_2 = \frac{V_x}{2\pi} = \frac{\pi^2}{4} \times \frac{1}{2\pi} = \frac{\pi}{8} \fallingdotseq 0.39$$

$y = \cos x$ より

$$\frac{dy}{dx} = -\sin x$$

$$\therefore dy = -\sin x \cdot dx$$

y	$0 \to 1$
x	$\frac{\pi}{2} \to 0$

$$V_y = \pi \int_0^1 x^2 \, dy$$

$$= \pi \int_{\frac{\pi}{2}}^0 x^2 \cdot \overbrace{(-\sin x)}^{(\cos x)'} \, dx$$

$$= \pi \cdot \left\{ \left[x^2 \cdot \cos x\right]_{\frac{\pi}{2}}^0 + \int_0^{\frac{\pi}{2}} 2x \cdot \overbrace{\cos x}^{(\sin x)'} \, dx \right\}$$

$$= \pi \left\{ 0 + \left[2x \cdot \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 2 \cdot \sin x \, dx \right\}$$

$$= \pi \left\{ \pi \cdot \sin \frac{\pi}{2} - 0 + \left[2 \cdot \cos x\right]_0^{\frac{\pi}{2}} \right\}$$

$$= \pi(\pi + 0 - 2) = \pi(\pi - 2)$$

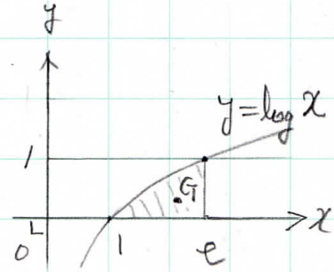
$$\therefore g_1 = \frac{V_y}{2\pi} = \frac{\pi(\pi - 2)}{2\pi} = \frac{\pi}{2} - 1 \fallingdotseq 0.57$$

(7)

$$S = \int_1^e \overset{x'}{1} \cdot \log x \, dx$$

$$= \left[x \cdot \log x\right]_1^e - \int_1^e x \cdot \frac{1}{x} \, dx$$

$$= e - \left[x\right]_1^e = 1$$



$y = \log x$ より

$$\frac{dy}{dx} = \frac{1}{x}$$

$$dx = x \cdot dy$$

x	$1 \to e$
y	$0 \to 1$

$$V_x = \pi \int_1^e y^2 \, dx$$

$$= \pi \int_0^1 y^2 \cdot \overset{(e^y)'}{x} \cdot dy$$

$$= \pi \left\{ \left[y^2 \cdot e^y\right]_0^1 - \int_0^1 2y \cdot \overset{(e^y)'}{e^y} \, dy \right\}$$

$$= \pi \left\{ e - \left(\left[2y \cdot e^y\right]_0^1 - \int_0^1 2 \cdot e^y \, dy \right) \right\}$$

$$= \pi \left\{ e - 2e + 2\left[e^y\right]_0^1 \right\}$$

$$= \pi \left\{ -e + 2(e-1) \right\} = (e-2)\pi$$

$$\therefore g_2 = \frac{(e-2)\pi}{2\pi} = \frac{e-2}{2}$$

$$\fallingdotseq 0.36$$

$$V_y = \pi e^2 - \pi \int_0^1 x^2 \, dy$$

$$= \pi e^2 - \pi \int_1^e x^2 \cdot \frac{1}{x} \, dx$$

$$= \pi e^2 - \pi \left[\frac{1}{2} x^2\right]_1^e$$

$$= \pi e^2 - \pi \left(\frac{1}{2} e^2 - \frac{1}{2}\right)$$

$$= \frac{e^2+1}{2} \pi$$

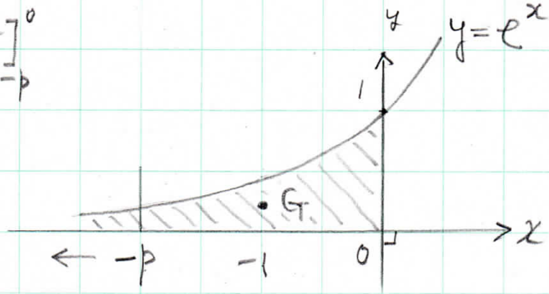
$$\therefore g_1 = \frac{e^2+1}{2} \pi \times \frac{1}{2\pi}$$

$$= \frac{e^2+1}{4}$$

$$\fallingdotseq 2.10$$

(8) $S = \lim_{p\to\infty} \int_{-p}^{0} e^x \, dx = \lim_{p\to\infty} \left[e^x \right]_{-p}^{0}$

$= \lim_{p\to\infty} \left(1 - \frac{1}{e^p}\right) = 1$



$V_x = \lim_{p\to\infty} \pi \int_{-p}^{0} (e^x)^2 dx = \lim_{p\to\infty} \pi \int_{-p}^{0} e^{2x} dx$

$= \lim_{p\to\infty} \pi \left[\frac{1}{2} e^{2x}\right]_{-p}^{0} = \lim_{p\to\infty} \frac{\pi}{2}\left(1 - \frac{1}{e^{2p}}\right) = \frac{\pi}{2}$

$\therefore g_2 = \frac{\pi}{2} \times \frac{1}{2\pi} = \frac{1}{4} = 0.5$

$y = e^x$ より

$\frac{dy}{dx} = e^x$

$dy = e^x \cdot dx$

y	$0 \to 1$
x	$-p \to 0$

$(p \to \infty)$

$V_y = \pi \int_0^1 x^2 dy = \lim_{p\to\infty} \pi \int_{-p}^{0} x^2 \cdot e^x \cdot dx$

$= \lim_{p\to\infty} \pi \cdot \left\{ \left[x^2 \cdot e^x\right]_{-p}^{0} - \int_{-p}^{0} 2x \cdot e^x dx \right\}$

$= \lim_{p\to\infty} \pi \left\{ 0 - \frac{p^2}{e^p} - \left[2x \cdot e^x\right]_{-p}^{0} + \int_{-p}^{0} 2 \cdot e^x dx \right\}$

$= \lim_{p\to\infty} \pi \left\{ \frac{2p}{e^p} + \left[2 \cdot e^x\right]_{-p}^{0} \right\}$

$= \lim_{p\to\infty} \pi \left(2 - \frac{2}{e^p}\right) = 2\pi$

※途中 $\lim_{p\to\infty} \frac{p^2}{e^p} = \lim_{p\to\infty} \frac{2p}{e^p} = \lim_{p\to\infty} \frac{2}{e^p} = 0$

$\therefore |g_1| = \frac{2\pi}{2\pi} = 1$, $g_1 = -1$

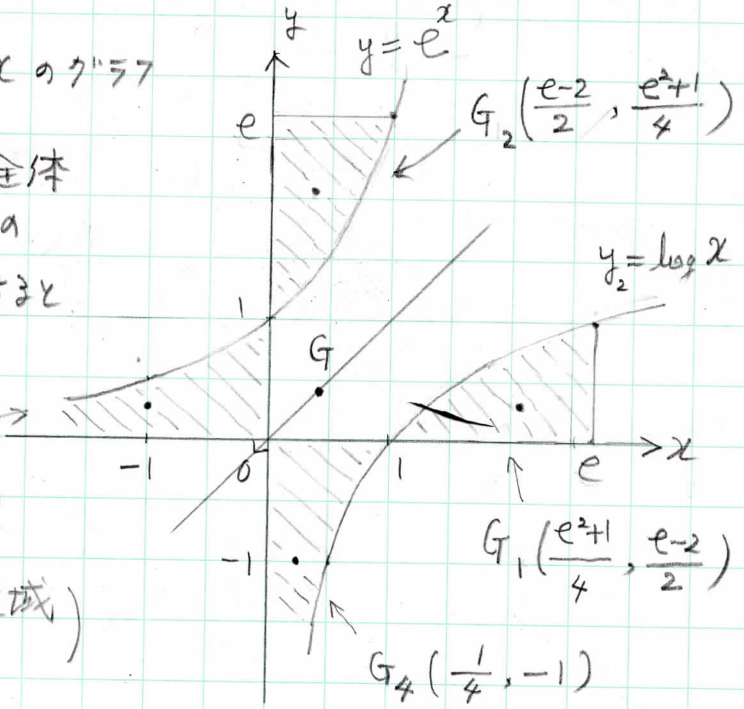
(7)、(8) より

$y_1 = e^x$, $y_2 = \log x$ のグラフ

で作られる斜線部全体

（面積は、計 4）の

重心を $G(g, g)$ とすると



（$G_{1\sim4}$ は、それぞれの領域における重心。）

$$g = \frac{1}{4}\left(\frac{1}{4} - 1 + \frac{e-2}{2} + \frac{e^2+1}{4}\right)$$

$$= \boxed{\frac{1}{16}(e^2 + 2e - 6)} \fallingdotseq 0.427$$

$$= \frac{1}{16}\left\{(e+1)^2 - 7\right\}$$